Finding $51^{1/2}$ Using Number Groups <= 50

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On pages 92-94 of ***Dead Reckoning: Calculating Without Instruments***, an example is given of calculating the square root of 51. Although the text suggests that individual number groups should be kept no larger than 50 to ease multiplications and minimize ripple effects on neighboring groups, the example does have one number group of 54. The following demonstrates that it is just as valid to adjust quotients and remainders in the steps in order to limit number groups to the given suggestion of 50. The steps are the same as the book up to b_3, at which point the quotient and remainder is adjusted get a b_4 no larger than 50. The subsequent steps produce different number groups than in the book, but in the end provide the same result for $51^{1/2}$.

The closest two-digit root of 5100 is 71 with a remainder of 59 = 71 R 59

Root thus far: 71

$$b_0 = \frac{5900/2}{71} = 41 \text{ R } 39$$

71 | 41

Now we start the algorithm:

$$b_1 = \frac{3900 - 41^2/2}{71} = 43 \text{ R } 6.5$$

71 | 41 | 43

$$b_2 = \frac{650 - 41(43)}{71} = (-16 \text{ R } -23) = -16 \text{ R } 23$$

71 | 41 | 43 | -16

$$b_3 = \frac{2300 - (-16)41 - 43^2/2}{71} = 28 \text{ R } 43.5 = 29 \text{ R } -27.5$$

71 | 41 | 43 | -16 | 29

$$b_4 = \frac{-2750 - (29)41 - (-16)43}{71} = -(45 \text{ R } 56) = -(46 \text{ R } -15) = -46 \text{ R } 15$$

71 | 41 | 43 | -16 | 29 | -46

$$b5 = \frac{1500 - (-46)41 - (29)43 - (-16)^2/2}{71} = 28 \ R\ 23$$

71 | 41 | 43 | -16 | 29 | -46 | 28

$$b6 = \frac{2300 - (28)41 - (-46)43 - (-16)29}{71} = 50 \ R\ 44$$

71 | 41 | 43 | -16 | 29 | -46 | 28 | 50

which is the point where the book stops the calculation.

Melding groups and adjusting the decimal point yields $51^{1/2}$ = 7.141428428542850…, which is the same approximation as in the book, and accurate to the last digit calculated.