

Mnemonics for Squares and Cubes of Two-Digit Numbers

Update Version: September 6, 2006

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As a tool for mental calculation, I have never been particularly fond of mnemonics, that is, methods for memorizing numbers by word association or letter patterns. Perhaps I would be if I were a performer called upon regularly to recall numbers or items, but my interests lie more along the lines of calculations one might encounter in the course of daily life. Since these can be varied and infrequent, I have a bent toward general rather than particular methods. That said, I found a set of mnemonics from a book published in 1910 to be a refreshing and fun way to quickly call to mind cubes of two-digit numbers, which is a sufficiently common task in the general methods to warrant an exception. The book is titled *Magician's Tricks: How They are Done*, by Henry Hatton and Adrian Plate, and the chapter that contains their mnemonic system can be found at <http://stepanov.lk.net/mnemo/mgtr.html>. In this paper, I have extended their scheme to provide squares as well as cubes of two-digit numbers, as these are so important for mental calculation. In addition, I have updated a number of their mnemonic phrases to ones using more modern terms, or to ones I think are an improvement over the original phrases.

Why Squaring is So Important in Mental Calculation

The ability to quickly find the square of a number is critical for fast mental calculation. Consider these applications:

1. The straightforward calculation of a square is a common task in mental arithmetic.
2. One of the most powerful tools in mental calculation is converting the multiplication of two different numbers into the square of the average minus the square of the difference. This is shown by the algebraic identity:

$$(a+c)(a-c) = a^2 - c^2$$

where **a** is the average of the two numbers, **(a+c)** is one of the numbers, and **(a-c)** is the other number. This is a very common technique in mental calculation. For example, $28 \times 32 = 30^2 - 2^2$, and $53 \times 77 = 65^2 - 12^2$. Less convenient multipliers can be manipulated in a number of ways to use this technique. Here are a couple of examples:

$$\begin{aligned} 28 \times 33 &= 28 \times 32 + 28 = 30^2 - 2^2 + 28 \\ \text{or} &= 28 \times 34 - 28 = 31^2 - 3^2 - 28 \\ 23 \times 67 &= 23(100-33) = 2300 - 23 \times 33 = 2300 - (28^2 - 5^2) \\ \text{or} &= 23 \times 33 \times 2 + 23 = 2(28^2 - 5^2) + 23 \\ \text{or} &= 23(50+17) = 2300/2 + 23 \times 17 = 2300/2 + 20^2 - 3^2 \end{aligned}$$

In the end, we reduce all the possible combinations of two-digit by two-digit multiplications to just the two-digit squares plus minor arithmetic adjustments.

3. Multiplication of two three-digit numbers can be converted as above: $244 \times 376 = 310^2 - 66^2$. But 310^2 is really just a two-digit square followed by two zeros—what if we end up with a three-digit square here? Then we use the identity in #2 above *in reverse*, so we split the square into two numbers equidistant from the original number, *adding* the square of that distance. Consider $244 \times 382 = 313^2 - 69^2 = [300(326) + 13^2] - 69^2$ and we end up with a simple calculation if we know the two-digit squares. Remember, the average squared is greater than the split numbers multiplied, so if you square the average of two numbers, you *subtract* the difference squared; if you split a square into two equidistant numbers multiplied, you *add* the difference squared. Squares of three-digit or four-digit numbers can use the expansion $(a+b)^2 = a^2 + b^2 + 2ab$, where the $2ab$ multiplication can again be simplified using squares. If the number has three digits, the middle digit can be included in either **a** or **b**, depending on which squares the calculator is most comfortable using.
4. Calculating square roots involves finding squares of intermediate solutions. This is true not only for the classic square root method, but also for alternatives based on Newton's method or other methods more suited to mental calculation, as described in my book, *Dead Reckoning: Calculating Without Instruments*. Calculating cube roots by taking weighted averages, described in the book, also requires squaring operations.
5. Power series formulas for functions such as logarithms and exponentials involve raising numbers to powers. For mental calculation, terminating the series at a power of 2 usually provides satisfactory precision. One example of using a power series that includes the squared term can be found in my paper on mentally calculating exponentials at http://www.myreckonings.com/Dead_Reckoning/Chapter_4/Materials/Bemer_Exponentials.pdf

These considerations have led me to revise my original opinion of mnemonics. I think mnemonics can be useful to a mental calculator as a fun tool for learning squares and as a more permanent method for producing cubes. However, there is no good reason to remember mnemonics for squares that are easily found in other ways, and there are very good reasons to learn the general strategies. Therefore,

- There are no entries in the table for multiples of 10, since $30^2 = 3^2 \times 100$ and $30^3 = 3^3 \times 1000$.
- Squares of numbers in the ranges 11-22 and 91-99, and all numbers ending in 5, are listed with a shortcut formula based on the reverse of the identity in #2 above. We split the square into two numbers each equidistant from the original number, multiply them, and then add the distance squared. The square of 13, for example, can be converted to $10 \times 16 + 3^2$, which is easily calculated. Above 15 we can use either 10 or 20 as the convenient multiplier: $18^2 = 10 \times 26 + 8^2$ or $16 \times 20 + 2^2$, but I prefer using 20. We really could have extended this to 29, and indeed all the squares can be split to a nearby multiple of 10, but the mnemonics can also be used here. The squares of numbers in the 90's all use 100 as a convenient multiplier. Squares of numbers ending in 5 are split into the multiples of 10 on either side, with 5^2 added—since both multipliers end in 0, we can just multiply the first digit by the first digit plus one, then append 25: $65^2 = (6 \times 7) | 25 = 4225$.
- Squares of numbers in the range 41-59 are listed with a different shortcut formula. For numbers near 50, we can add the difference from 50 to 25, multiply by 100, and add the difference squared. Since the distance is within 10 in the range 41-59, we can add the difference to 25 and simply append the distance squared. Here $53^2 = (25+3) | 3^2 = 2809$ and $44^2 = (25-6) | 6^2 = 1936$.

The Mnemonic Alphabet for Digits

This mnemonic scheme uses consonant sounds to replace digits according to the table below.

The way to initially remember this table is:

- t has one down-stroke
- n has two down-strokes
- m has three down-strokes
- r is the last letter of four
- L in Roman notation is fifty
- J looks something like a reversed six
- k, inverted, is similar to seven
- f in script resembles eight
- p is similar to a flipped nine
- c is the first letter of cipher, which is the word for naught.

Digit	Consonant Sound
1	t, d, th
2	n
3	m
4	r
5	l
6	j, ch, sh, zh, z as in azure, soft g as in genius
7	k, hard c, q, hard g, ing
8	f, v
9	p, b
0	s, z, soft c

It is important to treat these as sounds, not letters—this will help you remember the rest of the entries in the table.

The vowels a, e, i, o and u, and w, h, y are merely used to form words, as nail (n-l) = 25 and chess (ch-s) = 60. Silent letters, those that are not pronounced, have no value, as for example, knife (n-f) = 28; lamb (l-m) = 53; gh in thought; l in palm; and r in tapeworm. Double consonants are treated as one letter, as mummy (m-m) = 33 and butter (b-t-r) = 914, but if the double letters have distinct articulation, then each letter has its own numerical value, as accept (k-s-p-t) = 7091 and bookkeeper (b-k-k-p-r) = 97794. Since a whole number doesn't begin with zero, the letter s may be used at the beginning of the mnemonic to form a better phrase.

The idea here is that the keyword for the two-digit number will come to mind from the alphabet, and then the phrase associated with that keyword for the square or cube will be pictured, and then the result read out left-to-right from the alphabet sounds of the phrase.

Note that this mnemonic alphabet is identical to that presented as an aid for remembering intermediate results of mental calculations in the excellent books by Arthur Benjamin. However, the reference given earlier in this paper to the book published in 1910 makes it clear that this particular scheme was used at least as far back as the famous magician Harry Kellar.

So now we begin. The first column below is the original number, the second column is the square of the number, and the third column is the cube of the number. The starting numbers for the 1,000's, 10,000's, and 100,000's are noted in the table so that the proper grouping can be spoken as the number is read out from left to right. Oh, and please email me at ron@myreckonings.com if you come up with any better mnemonic phrases for a newer version of this paper—I'd love to hear them.

Mnemonics for Squares and Cubes of Two-Digit Numbers

3 Ham	9	An egg	2 7	
4 Rye	16	Sherry	6 4	
5 Lie	25	Denial	1 2 5	
6 Hash	36	On a dish	2 1 6	
7 Key	49	My room	3 4 3	
8 Hive	64	Wild honey	5 1 2	
9 Bee	81	Go nip	7 2 9	
11 Date	$10 \times 12 + 1^2 = 121$	With my mate	1 3 3 1	___ Begin 1,000's for cubes at 11
12 Dine	$10 \times 14 + 2^2 = 144$	Take enough	1 7 2 8	
13 Item	$10 \times 16 + 3^2 = 169$	Notebook	2 1 9 7	
14 Author	$10 \times 18 + 4^2 = 196$	Ink hirer	2 7 4 4	
15 Tell	$(1 \times 2) 25 = 225$	Me meekly	3 3 7 5	
16 Ditch	$20 \times 12 + 4^2 = 256$	Rosebush	4 0 9 6	
17 Talk	$20 \times 14 + 3^2 = 289$	Rap time	4 9 1 3	
18 Thief	$20 \times 16 + 2^2 = 324$	I love money	5 8 3 2	
19 Daub	$20 \times 18 + 1^2 = 361$	Save a lip	6 8 5 9	
21 Hand	$20 \times 22 + 1^2 = 441$	Punched	9 2 6 1	
22 Nun	$20 \times 24 + 2^2 = 484$	Does show her vow	1 0 6 4 8	___ Begin 10,000's for cubes at 22
23 Name	Will nab	A dandy joke	5 2 9 1 2 1 6 7	
24 New Year	Lucky age	With my fine rye	5 7 6 1 3 8 2 4	
25 Nile	$(2 \times 3) 25 = 625$	The lush Nile	6 7 6 1 5 6 2 5	
26 Wench	Watch cash	Took all cash	1 7 5 7 6	
27 Nag	Go nap	To buy each wife a home	7 2 9 1 9 9 6 8 3	
28 Enough	Giver	Neat plan	7 8 4 2 1 9 5 2	
29 Nip	Ferret	In arm of boy	8 4 1 2 4 3 8 9	
31 Mad	Pushed	In a big pout	9 6 1 2 9 7 9 1	
32 Man	Thy sinner	Many catch a wife	1 0 2 4 3 2 7 6 8	___ Begin 1,000's for squares at 32
33 My Home	Do save up	Home will be a hammock	1 0 8 9 3 5 9 3 7 1 1 5 6 3 9 3 0 4	

34 More	Do tell a wish	May be a miser 4 2 8 75
35 Mall	(3x4) 25 = 1225 1 2 9 6	Hurry in a vehicle 4 6 6 5 6
36 Smash	Stone bash 1 3 6 9	Rich jewel show 5 0 6 5 3
37 Smoke	Dumb chap 1 4 4 4	Lose a chilly home 5 4 8 7 2
38 Move	Steer rear 1 5 21	Lower a heavy can 5 9 3 1 9
39 My pay	To lend	Help me to buy 6 8 9 21
41 Road	(25-9) 00 + 9 ² = 1681	Chief point 7 4 0 8 8
42 Run	(25-8) 00 + 8 ² = 1764	Across a five 7 9 5 0 7
43 Rome	(25-7) 00 + 7 ² = 1849	Keep losing 8 5 1 8 4
44 Rower	(25-6) 00 + 6 ² = 1936	Awful diver 9 1 1 2 5
45 Rail	(4x5) 25 = 2025	A bad tunnel 9 7 3 3 6
46 Rich	(25-4) 00 + 4 ² = 2116	Big mummy show 1 0 3 8 2 3
47 Rake	(25-3) 00 + 3 ² = 2209	Hits my wife numb 1 1 0 5 9 2
48 Rough	(25-2) 00 + 2 ² = 2304	The wide sea will open 1 1 7 6 4 9
49 Rope	(25-1) 00 + 1 ² = 2401	To take a chair up 1 3 2 6 5 1
51 Lad	(25+1) 00 + 1 ² = 2601	The man child 1 4 0 6 0 8
52 Lion	(25+2) 00 + 2 ² = 2704	Dares chase a foe 1 4 8 8 7 7
53 Lamb	(25+3) 00 + 3 ² = 2809	Drove off a cook 1 5 7 4 6 4
54 Liar	(25+4) 00 + 4 ² = 2916	Idol crusher 1 6 6 3 7 5
55 Lily	(5x6) 25 = 3025	Dutch show my equal 1 7 5 6 1 6
56 Latch	(25+6) 00 + 6 ² = 3136	Tack a latch to a hatch 1 8 5 1 9 3
57 Look	(25+7) 00 + 7 ² = 3249	The evil tapeworm 1 9 5 1 1 2
58 Loaf	(25+8) 00 + 8 ² = 3364	Double the weight now 2 0 5 3 7 9
59 Lube	(25+9) 00 + 9 ² = 3481 3 7 2 1	When I sell my coupe 2 2 6 9 8 1
61 Shed	My wagon hut 3 8 4 4	An inch by a foot 2 3 8 3 2 8
62 Jane	My fairer 3 9 6 9	Name of my new wife 2 5 0 0 4 7
63 Gym	Wimpy shape 4 0 9 6	Any loss is work 2 6 2 1 4 4
64 Chair	Why raise? Push	Now a china drawer 2 7 4 6 2 5
65 Shallow	(6x7) 25 = 4225 4 3 5 6	Niagara channel 2 8 7 4 9 6
66 Judge	Harm wily witch 4 4 8 9	No fake rubbish 3 0 0 7 6 3
67 Joke	Rare fib 4 6 2 4	Amuse a sick chum 3 1 4 4 3 2

___ Begin 100,000's for cubes at 47

68 Shave	Your chin raw 4 7 6 1	May try your man 3 28 50 9
69 Shop	Here, cash it 5 0 4 1	Woman feels happy 3 57 9 1 1
71 Cat	Loose-eared 5 1 8 4	Milk by the day 3 7 3 2 4 8
72 Coin	Will die for 5 3 2 9	Make men rave 3 8 9 01 7
73 Game	Wily man up 5 47 6	Move past GO 4 0 5 2 2 4
74 Choir	Larkish	Rose Hill nunnery 4 2 1 8 7 5
75 Kill	(7x8) 25 = 5625 5 7 7 6	Run, thief—I kill! 4 3 8 9 7 6
76 Cage	Lock cage 5 9 2 9	Roomy if ape cage 4 5 6 53 3
77 Cook	Sell pan pie 6 0 8 4	Relish a lime ham 4 74 55 2
78 Give	Choose for 6 2 4 1	Here, a cruel loan 49 3 0 3 9
79 Cop	Shine red 6 5 6 1	Rip a museum up 5 3 1 4 4 1
81 Food	Chill shad 6 7 2 4	Well-made rare tea 5 51 3 6 8
82 Fan	Watch a hockey winner 6 8 8 9	Loyal to my Chevy 5 7 1 7 8 7
83 Foam	Shave if happy 7 0 5 6	Liquid quaffing 59 2 7 0 4
84 Fire	Goes hellish	We'll ban gas here 6 1 4 1 2 5
85 Fall	(8x9) 25 = 7225 7 3 9 6	Shatter the new well 6 3 6 0 5 6
86 Fish	Camp chow 75 6 9	Wish me a juicy leech 6 5 8 50 3
87 Fake	Kluge by 7 7 4 4	Jello fools me 6 8 1 4 7 2
88 Five	Gang warrior 7 9 2 1	Shaved a raccoon 7 0 49 6 9
89 VIP	Cabinet	Guess our bishop 75 3 5 7 1
91 Piety	$100 \times 82 + 9^2 = 8281$	Gloomy lookout 7 7 86 8 8
92 Pony	$100 \times 84 + 8^2 = 8464$	Kick a fish off a hive 8 0 4 3 5 7
93 Poem	$100 \times 86 + 7^2 = 8649$	"Wi-Fi" is rhyme-like 8 3 05 8 4
94 Bar	$100 \times 88 + 6^2 = 8836$	Famous loafer 8 5 7 3 7 5
95 Pill	(9x10) 25 = 9025	Vile chemical 8 8 4 7 3 6
96 Patch	$100 \times 92 + 4^2 = 9216$	Favoring a match 9 1 2 6 7 3
97 Pack	$100 \times 94 + 3^2 = 9409$	Beaten each game 9 41 1 9 2
98 Puff	$100 \times 96 + 2^2 = 9604$	Part the bun 9 7 0 2 9 9
99 Pup	$100 \times 98 + 1^2 = 9801$	Big sunny puppy