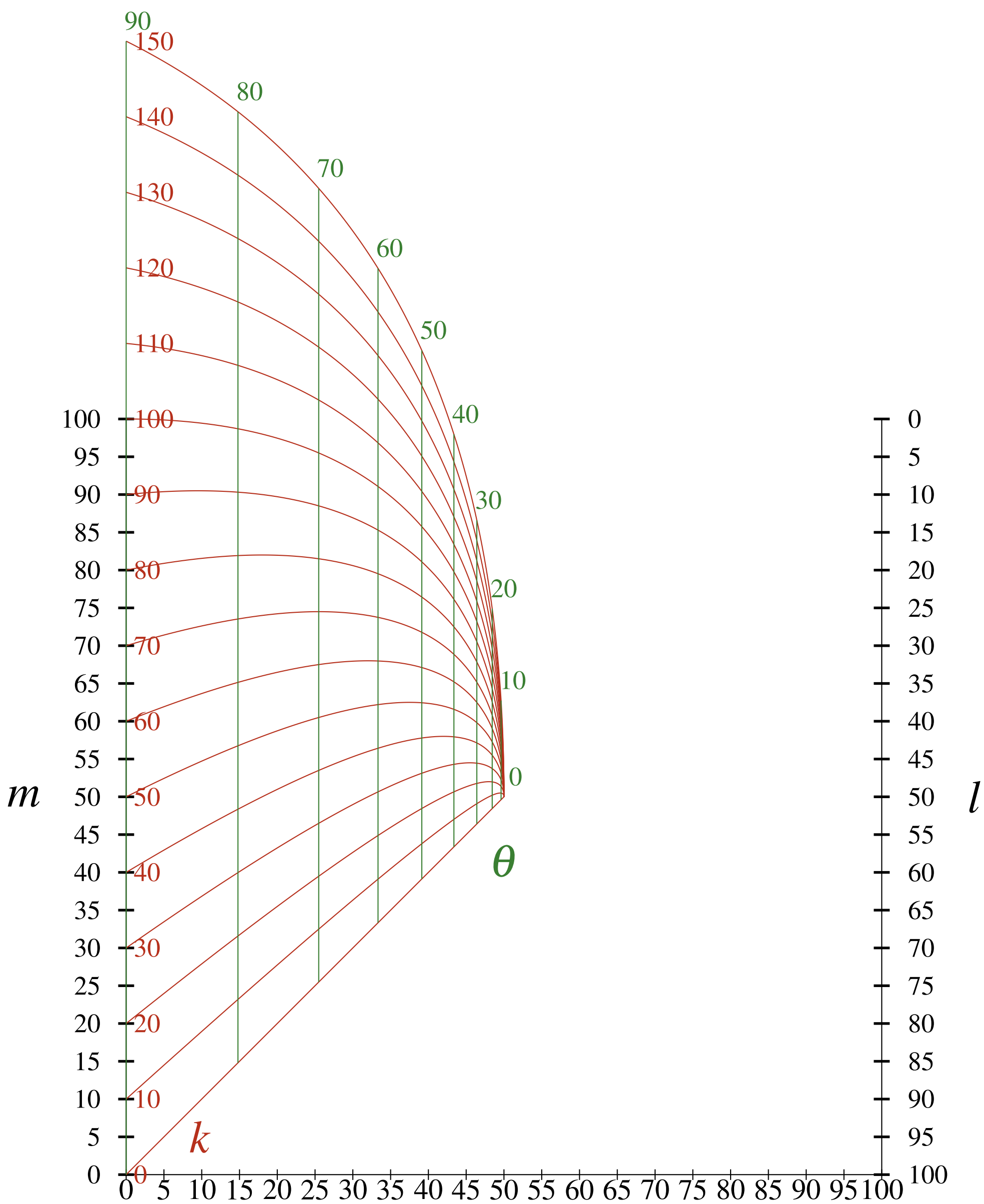
Transformation Exercises on a 4-Variable Nomogram (March 30, 2008)

(See http://myreckonings.com/wordpress/2008/03/13/a-4-variable-nomogram-%e5%9b%9b%e5%8f%98%e9%87%8f%e8%af%ba%e6%a8%a1%e5%9b%be/)

The original nomogram below has scale compression near $\theta = 0$ that leads to inaccurate results
(Note that this nomogram can be clipped above l=m=100)

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \begin{vmatrix} 0 & m & 1 \\ \frac{100\cos\theta}{1+\cos\theta} & \frac{k\sin\theta+100\cos\theta}{1+\cos\theta} & 1 \\ 100 & 100-l & 1 \end{vmatrix} = 0$$



$$m = k\sin\theta + l\cos\theta$$

Transformation of Original Nomogram from A.S Hall, **Construction of Graphs and Charts** (1958)

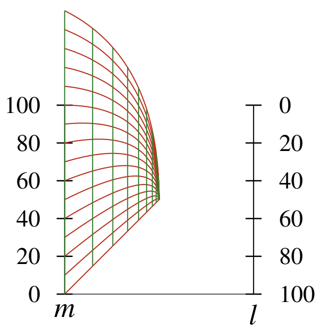
$$\begin{vmatrix} \frac{x_1}{(1-b)x_1+b} & \frac{b(y_1+ax_1)}{(1-b)x_1+b} & 1 \\ \frac{x_2}{(1-b)x_2+b} & \frac{b(y_2+ax_2)}{(1-b)x_2+b} & 1 \\ \frac{x_3}{(1-b)x_3+b} & \frac{b(y_3+ax_3)}{(1-b)x_3+b} & 1 \end{vmatrix} = 0$$

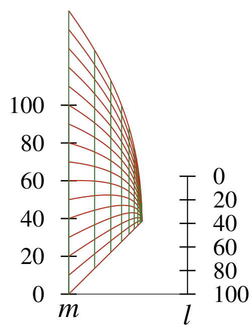
Diagram order: (Note that $a = 0, b = 1$ provides the original nomogram)

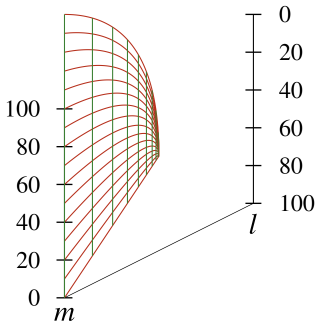
$a = 0, b = 1$ $\qquad$ $a = 0, b = 0.994$

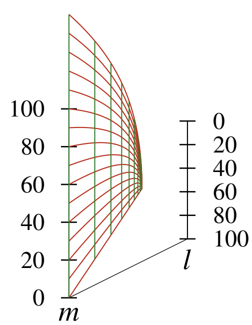
$a = 0.5, b = 1$ $\qquad$ $a = 0.5, b = 0.994$

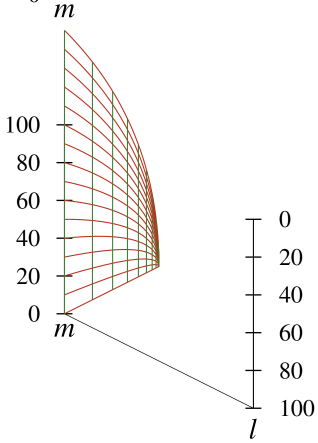
$a = -0.5, b = 1$ $\qquad$ $a = -0.5, b = 0.994$

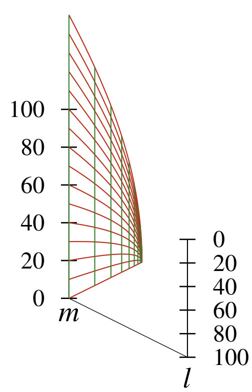








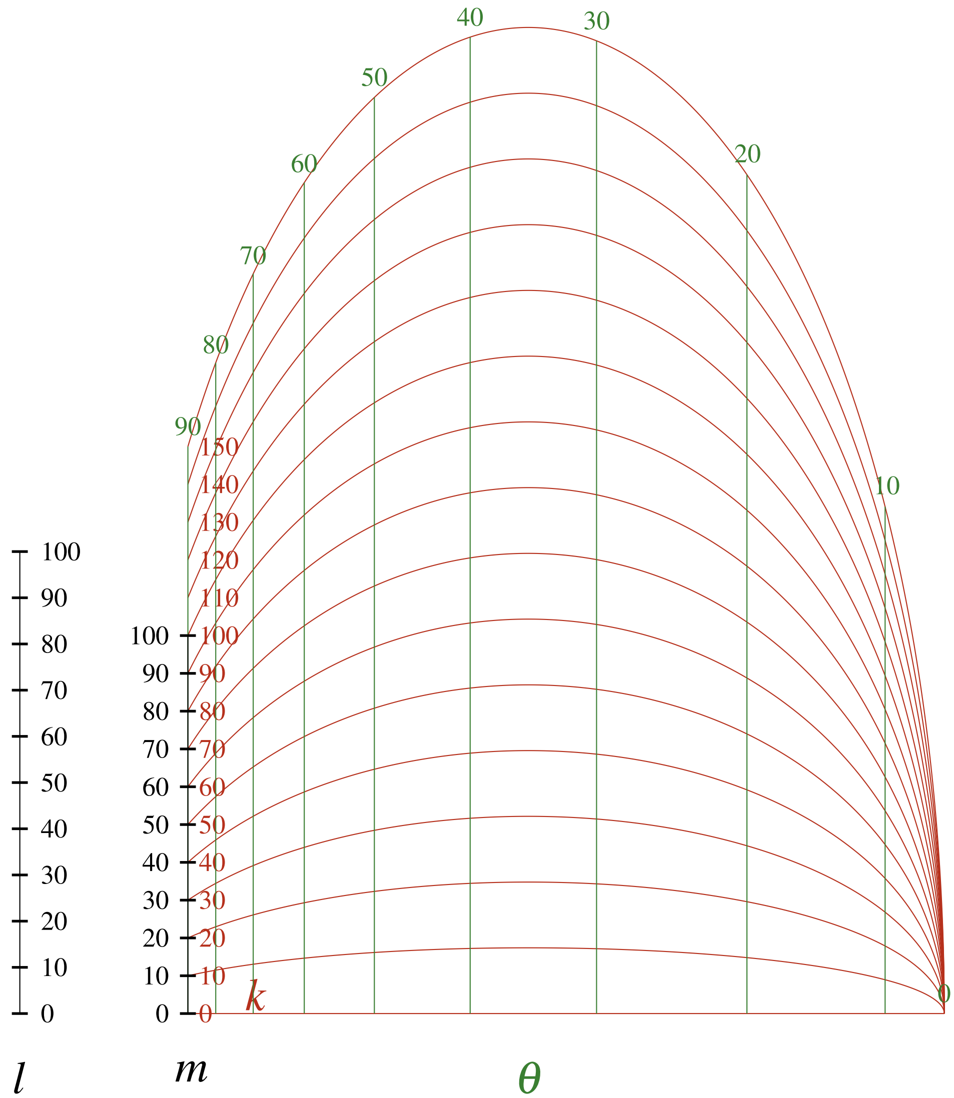




These transformations seem to be useful to square up a nomogram, but the original nomogram is already square.

Central Projection Transformation of Original Nomogram through $P = (x_p, y_p, z_p) = (55, 55, -20)$

$$\begin{vmatrix} \frac{z_p x_1}{x_1 - x_p} & \frac{y_p x_1 - x_p y_1}{x_1 - x_p} & 1 \\ \frac{z_p x_2}{x_2 - x_p} & \frac{y_p x_2 - x_p y_2}{x_2 - x_p} & 1 \\ \frac{z_p x_3}{x_3 - x_p} & \frac{y_p x_3 - x_p y_3}{x_3 - x_p} & 1 \end{vmatrix} = 0$$

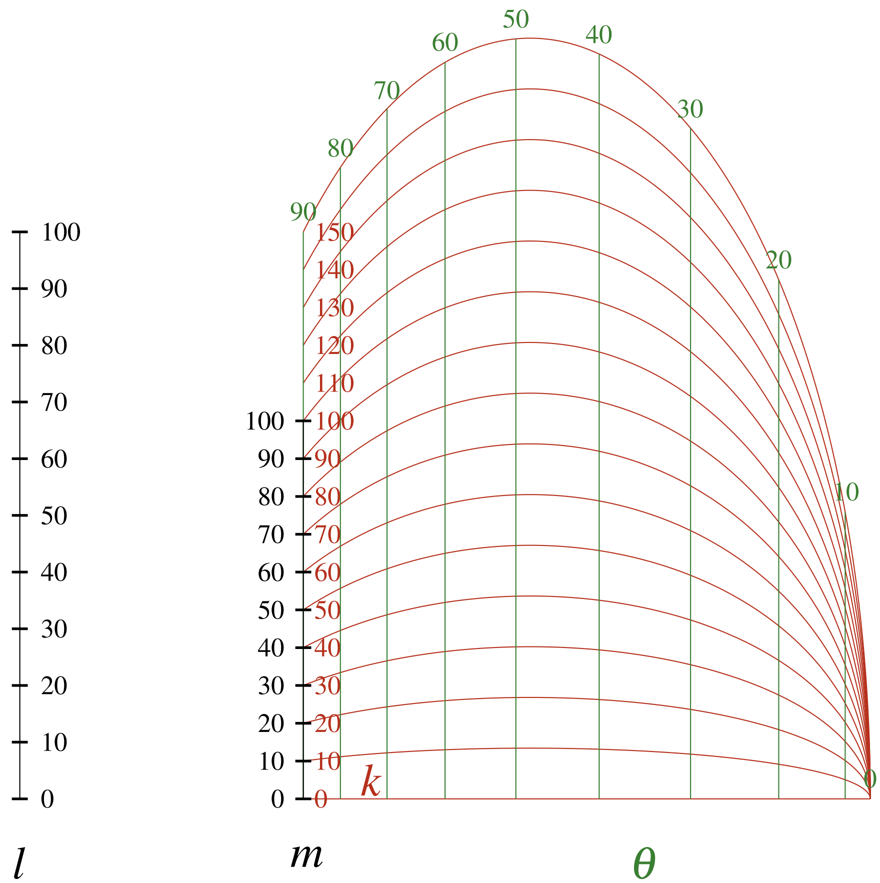


$$m = k \sin\theta + l \cos\theta$$

This projective transformation is very effective in spreading out the grid more evenly, but the l-scale and m-scale have become much smaller. Also, the needed height for the chart (everything below a line passing through l=0 and m=100) is also much higher than before, limiting the vertical enlargement that would spread out the scales.

Central Projection Transformation of Original Nomogram through $P = (x_p, y_p, z_p) = (60, 60, -30)$

$$\begin{vmatrix} \frac{z_p x_1}{x_1 - x_p} & \frac{y_p x_1 - x_p y_1}{x_1 - x_p} & 1 \\ \frac{z_p x_2}{x_2 - x_p} & \frac{y_p x_2 - x_p y_2}{x_2 - x_p} & 1 \\ \frac{z_p x_3}{x_3 - x_p} & \frac{y_p x_3 - x_p y_3}{x_3 - x_p} & 1 \end{vmatrix} = 0$$



$$m = k \sin\theta + l \cos\theta$$

This projective transformation may be the most effective one, since the l-scale is not reduced as much as in the previous nomogram and the needed height for the chart is also less, allowing a greater vertical enlargement.