Transformation Exercises on a 4-Variable Nomogram (March 30, 2008)

(See http://myreckonings.com/wordpress/2008/03/13/a-4-variable-nomogram-%e5%9b%9b%e5%8f%98%e9%87%8f%e8%ad%a6%e6%a8%a1%e5%9b%be/)

The original nomogram below has scale compression near $\theta = 0$ that leads to inaccurate results
(Note that this nomogram can be clipped above $l=m=100$)

\[
\begin{align*}
\begin{array}{c|c|c|c|c|c|c}
 x_1 & y_1 & 1 & 0 & \frac{m}{100\cos \theta} & \frac{m}{1 + \cos \theta} & 1 \\
x_2 & y_2 & 1 & \frac{m}{100\cos \theta} & \frac{k \sin \theta + 100 \cos \theta}{1 + \cos \theta} & 1 & 0 \\
x_3 & y_3 & 1 & 1 & 100 - l & 1 \\
\end{array}
\end{align*}
\]

\[m = k \sin \theta + l \cos \theta\]

\[
\begin{array}{ccc}
x_1 & b(x_1+ax_1) & 1 \\
(1-b)x_1+b & (1-b)x_1+b & 1 \\
x_2 & b(x_2+ax_2) & 1 \\
(1-b)x_2+b & (1-b)x_2+b & 1 \\
x_3 & b(x_3+ax_3) & 1 \\
(1-b)x_3+b & (1-b)x_3+b & 1 \\
\end{array}
\]

\[= 0\]

Diagram order: (Note that \(a = 0, b = 1\) provides the original nomogram)

- \(a = 0, b = 1\)
- \(a = 0.5, b = 1\)
- \(a = -0.5, b = 1\)
- \(a = 0, b = 0.994\)
- \(a = 0.5, b = 0.994\)
- \(a = -0.5, b = 0.994\)

These transformations seem to be useful to square up a nomogram, but the original nomogram is already square.
Central Projection Transformation of Original Nomogram through $P = (x_p, y_p, z_p) = (55, 55, -20)$

\[
\begin{vmatrix}
\frac{z_p x_1}{x_1 - x_p} & \frac{y_p x_1 - x_p y_1}{x_1 - x_p} & 1 \\
\frac{z_p x_2}{x_2 - x_p} & \frac{y_p x_2 - x_p y_2}{x_2 - x_p} & 1 \\
\frac{z_p x_3}{x_3 - x_p} & \frac{y_p x_3 - x_p y_3}{x_3 - x_p} & 1
\end{vmatrix} = 0
\]

This projective transformation is very effective in spreading out the grid more evenly, but the $l$-scale and $m$-scale have become much smaller. Also, the needed height for the chart (everything below a line passing through $l=0$ and $m=100$) is also much higher than before, limiting the vertical enlargement that would spread out the scales.

\[m = k \sin \theta + l \cos \theta\]
Central Projection Transformation of Original Nomogram through \( P = (x_p, y_p, z_p) = (60, 60, -30) \)

\[
\begin{bmatrix}
\frac{z_p x_1}{x_1 - x_p} & \frac{y_p x_1 - x_p y_1}{x_1 - x_p} & 1 \\
\frac{z_p x_2}{x_2 - x_p} & \frac{y_p x_2 - x_p y_2}{x_2 - x_p} & 1 \\
\frac{z_p x_3}{x_3 - x_p} & \frac{y_p x_3 - x_p y_3}{x_3 - x_p} & 1
\end{bmatrix} = 0
\]

\[ m = k \sin \theta + l \cos \theta \]

This projective transformation may be the most effective one, since the \( l \)-scale is not reduced as much as in the previous nomogram and the needed height for the chart is also less, allowing a greater vertical enlargement.